

Maximum lifetime broadcasting problem in sensor networks

Z. Lipiński

Institute of Mathematics and Informatics
Opole University, Poland

November 19, 2015

Abstract

We solve the maximum lifetime problem for the point-to-point and point-to-multipoint broadcast data transmission in one dimensional regular sensor network. Based on the analytical solution of the problem for one dimension we propose an algorithm solving the maximum lifetime broadcasting problem for point-to-point data transmission for any dimension.

Key words: wireless communication, broadcast transmission, sensor network lifetime.

1 Introduction

A sensor network is type of a wireless ad-hoc network which nodes have limited power and computational resources. Typical activity of sensor network nodes is collection of sensed data, performing simple computational tasks and transmission of the resulting data to a fixed set of data collectors. These activities are energy consuming and require cooperation between nodes so they can share their energy and other resources. The sensors utilize most of their energy in the process of data transmission, this energy grows with the size of the network and the amount of data transmitted over the network. One of the most important problems in sensor networks is to optimize the energy consumed by the sensor network nodes to extend their operating time and extend the lifetime of the whole network. By a sensor network lifetime we mean the time until first node of the network runs out of energy, [1, 2]. If the batteries of the nodes have initial energy E_0 , then by finding the optimal energy utilization by each node E_i^{opt} we can determine the number of cycles $N_{\text{cycles}} = \lfloor \frac{E_0}{E_i^{\text{opt}}} \rfloor$ the network can perform its functions until the most overloaded node runs out of its energy E_i^{opt} .

In wireless networks one can distinguish two types of data transmissions: point-to-point and point-to-multipoint. In the point-to-point transmission the sender transmits data to a unique receiver. In the point-to-multipoint transmission the transmitter sends in parallel the same data to a set of receivers. Important feature of the point-to-multipoint data transmission is the wireless multicast advantage property (the WMA property) [3]. For such transmission the nodes, which are in the range of the transmitting node, can receive the data without additional cost for the transmitter. In this paper we analyze the maximum lifetime broadcasting problem for wireless ad-hoc networks in which nodes use the point-to-point and point-to-multipoint data transmission methods. We analyze the optimal broadcast data transmission in the one dimensional, regular sensor network L_N , for which the data transmission cost energy matrix $E(x_i, x_j)$ is an arbitrary super-additive function, i.e., it satisfies the inequality

$$\forall x_i \leq x_j \leq x_k \quad E(x_i, x_j) + E(x_j, x_k) \leq E(x_i, x_k), \quad (1)$$

where x_i is the location of the network nodes on the line R . The matrix $E(x_i, x_j)$ defines the transmission cost of one unit of data between i -th and j -th node. Based on the analytical solution of

the maximum lifetime broadcasting problem for L_N network, we propose a point-to-point broadcast data transmission algorithm which allows to maximize the lifetime of sensor networks in two and more dimensions.

Let us denote by S_N a one dimensional sensor network build of N nodes. The nodes are located at the points $x_i, i \in [1, N]$ of a real line. By Q_k we denote the amount of data which generates and broadcasts the k -th node. The data transmission flow matrix $q^k(x_i, x_j) \equiv q_{i,j}^k$ defines the amount of data which is sent by the i -th node to the j -th node in the process of data broadcasting. If the k -th node broadcasts the amount Q_k of data then the energy spent by the i -th node to retransmit this data is given by the formula

$$E_i(q^k) = \sum_{j=1, j \neq i}^N q_{i,j}^k E_{i,j}. \quad (2)$$

The requirement, that each node must receive Q_k of data can be written in the form

$$\forall_{j \in [1, N], j \neq k} \sum_{i=1, i \neq j}^N q_{i,j}^k = Q_k. \quad (3)$$

To solve the maximum lifetime broadcasting problem means to find a data transmission flow matrix $q_{i,j}^k$, such that the objective function

$$E_{i'}(q^k) = \max_i \{E_i(q^k)\}, \quad (4)$$

reaches its minimum. The maximum lifetime broadcasting problem, later referred to as MLB problem, is defined by the set of parameters: the number of nodes $|S_N| = N$, the amount of data Q_k which is broadcasted by the k -th node, $k \in [1, N]$ and the data transmission cost energy matrix $E_{i,j}$. If the matrix $E_{i,j}$ is a function of a distance $d(x_i, x_j)$ between two nodes, then instead of defining the matrix elements $E_{i,j}$ one can give the location of network nodes in the space and determine the values of $E_{i,j}$ from the metric $d(x_i, x_j)$, $i, j \in [1, N]$. In this paper, we assume that $E_{i,j}$ is an arbitrary polynomial function of the distance $d(x_i, x_j)$ and satisfies (1).

We represent a sensor network S_N as a directed, weighted graph $G_N = \{S_N, V, E\}$ in which S_N is a set of graph nodes, V is the set edges and E set of weights. Each directed edge $t_{i,j} \in V$ defines a communication link between i -th and j -th node of the network. To each edge $t_{i,j}$ we assign a weight $E_{i,j}$, which is the cost of transmission of one unit of data between i -th and j -th node. By $U_i^{(\text{out})} \subseteq S_N$ we denote a set of the network nodes to which the i -th node can sent the data

$$U_i^{(\text{out})} = \{x_j \in S_N \mid \exists t_{i,j} \in V\}.$$

The set $U_i^{(\text{out})}$ defines the maximal transmission range of the i -th node. In the paper we assume, that each node of S_N can send data to any other nodes of the network, i.e.,

$$\forall_{i \in [1, N]}, \quad U_i^{(\text{out})} = S_N. \quad (5)$$

The data flow in the network S_N we describe in terms of spanning trees of the graph G_N . The set of all spanning trees, which begin in the k -th node we denote by V_k . If the assumption (5) is satisfied, then G_N is a complete graph and the number of spanning trees rooted at k -th node $|V_k|$ it is equal to N^{N-2} , [4]. We will identify a tree $t^{k,r}$ with the set of its edges. Because each tree $t^{k,r}$ defines the data flow from the broadcasting node to all other nodes of the network then they must have a direction and must be ordered in a sequence that the data is transmitted along the tree. We assume, that edges of the tree $t^{k,r} \in V_k$ have the natural orientation from the root node to the root child, etc., and are arranged with some fixed, initial sequence

$$t^{k,r} = (t_{i_1, j_1}^{k,r}, \dots, t_{i_{N-1}, j_{N-1}}^{k,r}). \quad (6)$$

For a point-to-point data transmission the solution of maximum lifetime broadcasting problem does not depend on the ordering of the edges. The ordering is important for the point-to-multipoint broadcast data transmission, where different wireless multicast data transmissions are defined by different orderings of the edges in the tree $t^{k,r}$.

By $q_{i,j}^{k,r}$ we denote the amount of data which is transmitted along the edge $t_{i,j}^{k,r}$. The data $q_{i,j}^k$ which is sent by the i -th node to the j -th node along all trees is given by the formula

$$q_{i,j}^k = \sum_{r=1}^{N^{N-2}} q_{i,j}^{k,r} t_{i,j}^{k,r}. \quad (7)$$

We require, that along each tree the transmitted data is the same, which means that for fixed k and r the weights of the edges $t_{i,j}^{k,r}$ are equal, $\forall_{i,j,i',j'} q_{i,j}^{k,r} = q_{i',j'}^{k,r}$. We denote these weights by q_r^k , i.e.,

$$\forall_{i,j} q_{i,j}^{k,r} = q_r^k. \quad (8)$$

The objective function of the MLB problem (4) can be written in terms of trees $t^{k,r}$ and weights q_r^k with the help of the following formula for the energy of each sensor

$$E_i^k = \sum_{j=1, j \neq i, k}^N \sum_{r=1}^{N^{N-2}} q_r^k t_{i,j}^{k,r} E_{i,j}. \quad (9)$$

The requirement (3) written in terms of $t^{k,r}$ and q_r^k has the form

$$\sum_r q_r^k = Q_k. \quad (10)$$

With the help of (9) and (10) the maximum lifetime broadcasting problem can be defined by the set of the following formulas

$$\begin{cases} \min_{q^k} \max_{i \in L_N} \{E_i^k(q^k)\}_{i=1}^N, & k \in [1, N], \\ E_i^k(q^k) = \sum_{j=1}^N \sum_r q_r^k t_{i,j}^{k,r} E_{i,j}, & E_{i,j} \geq 0, \\ t^{k,r} \in V_k, & q_r^k \geq 0, \quad r \in [1, N^{N-2}], \\ \sum_r q_r^k = Q_k, \end{cases} \quad (11)$$

where $q^k = (q_1^k, \dots, q_{N^{N-2}}^k)$. A solution of the MLB problem (11), when the data is broadcasted by the k -th node, will be given by an optimal set of trees $\{t^{k,r}\}_{r=1}^{N'} \subset V_k$, $N' \leq N$, weighted by N' non-negative numbers $q_r^{k,\text{opt}}$, which satisfy (10) and minimize the objective function (4). The following lemma describes relation between the local minima of the objective function (4) and global solutions of (11).

Lemma 1. Any minimum of the objective function (4) is a solution of the maximum lifetime broadcasting problem.

Proof. The feasible set

$$U(q^k) = \{q^k \in R_+^{N^{N-2}} \mid \sum_{r=1}^{N^{N-2}} q_r^k - Q_k = 0\},$$

where R_+ is a set of non-negative real numbers and is a domain of the objective function $E(q^k)$. From the linearity of the set $U(q^k)$ it follows, that any two points $q_0^k, q_1^k \in U(q^k)$ can be connected by a line segment and $\forall_{t \in [0,1]} tq_0^k + (1-t)q_1^k \in U(q^k)$. The objective function (4) of the maximum lifetime broadcasting problem (11) is piecewise linear in the q^k variable, which means that $E(tq_0^k + (1-t)q_1^k) = tE(q_0^k) + (1-t)E(q_1^k)$. Let us assume, that q_0^k is a minimum of $E(q^k)$ and there exists

another local minimum q_1^k such that $E(q_0^k) < E(q_1^k)$, then from the linearity of $E(q^k)$ it follows that $\forall_{t \in [0,1]} E(tq_0^k + (1-t)q_1^k) = tE(q_0^k) + (1-t)E(q_1^k) < tE(q_1^k) + (1-t)E(q_1^k)$ and $\forall_{t \in [0,1]} q^k(t)$, the inequality $E(q^k(t)) < E(q_1^k)$ holds, which means that q_1^k cannot be a minimum of $E(q^k)$ in $U(q^k)$ and q_0^k is a global minimum. \diamond

If we assume that $Q_k, q_{i,j} \in Z_+$ in (11), then we get the mixed integer linear programming problem for the network lifetime. The following lemma describes the complexity of the problem.

Lemma 2. The maximum lifetime broadcasting problem is NP-hard.

Proof. We reduce the partition problem to the MLB problem (11). To solve the MLB problem we must find a set of weights $\{q_r^k\}_{r \in [1, N^{N-2}]}$ for all trees from V_k , which minimize the objective function (4). For the integer values of $q_r^k \in Z_0^+$, and $Q_k \in Z^+$ to find the weights $q_r^k, r \in [1, N^{N-2}]$ satisfying $\sum_r q_r^k = Q_k$ it is equivalent to solving the partition problem, [5]. \diamond

2 Solution of the MLB problem for L_N

In one dimension, the superadditivity property (1) of the data transmission cost energy matrix $E_{i,j}$ means that, the lowest cost of delivery of a given amount of data from the i -th node located at x_i to the j -th node located at x_j is reached for the next hop data transmission along the shortest path. By a shortest path we mean a distance between two nodes, $d(x_i, x_j) = |x_i - x_j|$. It can be easily shown that in one dimension for $E_{i,j} = |x_i - x_j|^a, x_i, x_j \in R_+$ the inequality (1) is satisfied for any $a \geq 1$. From the above it follows that any function of the form $E_{i,j}(\bar{a}, \bar{\lambda}) = \sum_{n=0}^{\infty} \lambda_n |x_i - x_j|^{a_n}$, where $\lambda_n \geq 0, a_n \geq 1$ also satisfies (1). Nodes of a regular network L_N are placed at the points $x_i = i$ of a line and the distance between neighboring nodes is equal to one, $d_{i,i+1} = 1$. We can assume, without loosing the generality of the problem (11) that, $E_{i,i+1}(\bar{a}, \bar{\lambda}) = 1$, which is equivalent to the requirement that $\sum_{n=0}^{\infty} \lambda_n = 1$. By E_r we denote the cost of transmission of one unit of data between nodes which distance is $d_{i,i+r} = r, i, r \in [1, N]$. The function E_r defines weight for all edges $t_{i,i+r}$ of a given transmission tree t in V_k and it has the form

$$E_r(\bar{a}, \bar{\lambda}) = \sum_{n=0}^{\infty} \lambda_n r^{a_n}, \quad r \in [1, N], \quad (12)$$

where $\sum_{n=0}^{\infty} \lambda_n = 1, \lambda_n \geq 0$ and $a_n \geq 1$. We show that for one dimensional, regular sensor network L_N with the data transmission cost energy matrix of the form (12) there exists an equal energy solution of the maximum lifetime broadcasting problem for nodes which are inside the network, i.e. $i \in [2, N-1]$, and non equal energy solution for the border nodes, $i = 1, N$ of L_N . The next lemma describes the solution of the MLB problem for the first and last node of the L_N network.

Lemma 3. The solution of the maximum lifetime broadcasting problem (11) with $E_{i,j}$ satisfying (1), for the first and last node of the L_N network is given by the transmission graphs $(t_{1,2}, \dots, t_{N-1,N})$ and $(t_{N,N-1}, \dots, t_{2,1})$ with the weights Q_1 and Q_N respectively.

Proof. From the superadditivity property (1) of the data transmission cost energy matrix $E_{i,j}$ it follows that, the nodes consume minimal energy when they send all of their data to the nearest node. Because the distance between neighboring nodes of the regular network L_N is the same, then the energy spent by each node to transmit a given amount of data to its neighbor is also the same. From the above it follows that, when the broadcasted data Q_1 is transmitted along the edges $t_{i,i+1}, i \in [1, N-1]$, then each node $i \in [1, N-1]$ spends the same, minimal amount of energy. This means that the optimal transmission, a solution to the MLB problem for the first node is given by the data transmission graph $(t_{1,2}, \dots, t_{i,i+1}, \dots, t_{N-1,N})$ with the weight Q_1 . In a similar way one can find the solution of (11) for the last node of the L_N network. It is given by the transmission graph $(t_{N,N-1}, \dots, t_{i,i-1}, \dots, t_{2,1})$ weighted with the Q_N of data. \diamond

The next theorem describes the solution of (11) for the one dimensional, regular sensor network L_N , when the data is broadcasted by the internal nodes, $k \in [2, N-1]$ of the network.

Theorem 1. The solution of the maximum lifetime broadcasting problem (11), with the data cost energy matrix of the form (12), when the data is broadcasted by the k -th node, $k \in [2, N-1]$, is an equal energy solution and it is given by the N transmission trees t^r , $r \in [1, N]$

$$\begin{aligned} & (t_{k,k-1}^r, \dots, t_{2,1}^r, t_{r,k+1}^r, t_{k+1,k+2}^r, \dots, t_{N-1,N}^r), \quad r \in [1, k-1], \\ & (t_{k,k-1}^k, \dots, t_{2,1}^k, t_{k,k+1}^k, \dots, t_{N-1,N}^k), \quad r = k, \\ & (t_{k,k+1}^r, \dots, t_{N-1,N}^r, t_{r,k-1}^r, t_{k-1,k-2}^r, \dots, t_{2,1}^r), \quad r \in [k+1, N], \end{aligned} \quad (13)$$

with the weights

$$q_r^k = \begin{cases} \frac{E_1}{E_k} q_k^k + \frac{E_1}{E_k} Q_k, & r = 1, \\ \frac{E_1}{E_{k+1-r}} q_k^k, & r \in [2, k-1], k \geq 3, \\ \frac{E_1}{E_{r-k+1}} q_k^k, & r \in [k+1, N-1], k \leq N-2, \\ \frac{E_1}{E_{N-k+1}} q_k^k + \frac{E_1}{E_{N-k+1}} Q_k, & r = N, \end{cases} \quad (14)$$

where

$$q_k^k = \frac{1 - \frac{E_1}{E_k} - \frac{E_1}{E_{N-k+1}}}{-1 + \sum_{i=1}^k \frac{E_1}{E_i} + \sum_{i=1}^{N-k+1} \frac{E_1}{E_i}} Q_k. \quad (15)$$

Proof. We assume that the formulas (13) and (14) are solution to the MLB problem for N and show that it is also true for $(N+1)$. After adding the $(N+1)$ -th node at the end of the network, the nodes from the L_N subnetwork can transmit data to the $(N+1)$ -th node by a trees $t^{r'}(L_{N+1})$, $r' \in [1, N']$ such, that $t^{r'}(L_{N+1})|_{L_N} = t^r$ and t^r are given by (13). Because all Q_k of data must be delivered to the $(N+1)$ -th node by the nodes from the L_N subnetwork, then for data transmission cost energy matrix satisfying (1) the lower costs of delivery of these data is when the N -th node passes all Q_k of data to the $(N+1)$ -th node by the 'next hop' data transmission, consuming the amount

$$E_N^k = E_1 Q_k$$

of energy. This means that the optimal trees $t^{r'}(L_{N+1})$ for data transmission from the L_N subnetwork are given by the ordered set of edges

$$t^r(L_{N+1}) = (t^r, t_{N,N+1}^r), \quad r \in [1, N], i, j \in [1, N], \quad (16)$$

where the edges of the t^r tree are given by (13).

Because, the $(N+1)$ -th node must take part in data retransmissions, we must find set of trees which begin in the k -th node and for which the $(N+1)$ -th node is not a leaf. From (16) follows that all Q_k of data is transmitted to the $(N+1)$ -th node in the L_N subnetwork by the next hop data transmission graph, starting from the $(k+1)$ -th node. It means that the first $(N+1-k)$ edges of the optimal trees $t^{r''}$, $r'' \in [N+1, N']$ are

$$t^{r''} = (t_{k,k+1}^{r''}, \dots, t_{N,N+1}^{r''}). \quad (17)$$

The nearest node to which the $(N+1)$ -th node can send the $q_{r''}^k$ data, without transmitting data in a loop, is the $(k-1)$ -th node, which means that next edge, which we must add to the set (17) is $t_{N+1,k-1}^{r''}$. The next edges in the transmission trees (17) are $t_{k-i,k-i+1}^{r''}$, $i \in [1, k-1]$, and define the most efficient data transmission for $E_{i,j}$ satisfying (1). From the above it follows, that the optimal data transmission tree for the $(N+1)$ -th node to retransmit part of the broadcasted data in the L_{N+1} network is given by the formula

$$t^{N+1} = (t_{k,k+1}^{N+1}, \dots, t_{N,N+1}^{N+1}, t_{N+1,k-1}^{N+1}, t_{k-1,k-2}^{N+1}, \dots, t_{2,1}^{N+1}), \quad (18)$$

which is the $(N+1)$ tree in (13) for the L_{N+1} network.

We show that it is enough to add one tree t^{N+1} to the solution of MLB problem in L_N network to obtain the set of trees which is a solution for L_{N+1} . For the set of trees $\{(t^r, t^{N+1})\}_{r=1}^N$ given by (13) and (18) and weights $\{(q_r^k, q_{N+1}^k)\}_{r=1}^N$ an equal energy requirement can be written in the form

$$E_i(q^k) = E_{i+1}(q^k), \quad i \in [1, N], \quad (19)$$

where

$$E_i(q^k) = \begin{cases} q_1^k E_k, & i = 1, \\ q_i^k E_{k+1-i} + Q_k E_1, & i \in [2, k], \\ q_i^k E_{i-k+1} + Q_k E_1, & i \in [k+1, N-1], k \leq N-1, \\ q_N^k E_{N-k+1} + Q_k E_1, & i = N, \\ q_{N+1}^k E_{N-k+2}, & i = N+1. \end{cases}$$

It is easy to check that the solution of the equations (19), with the requirement $\sum_{r=1}^{N+1} q_r^k = Q_k$, is given by (14) for the L_{N+1} network. \diamond

The Figure 1 shows the optimal set of N trees for a solution of the MLB problem in L_N when the data is broadcasted by the k -th node, $k \in [2, N-1]$.

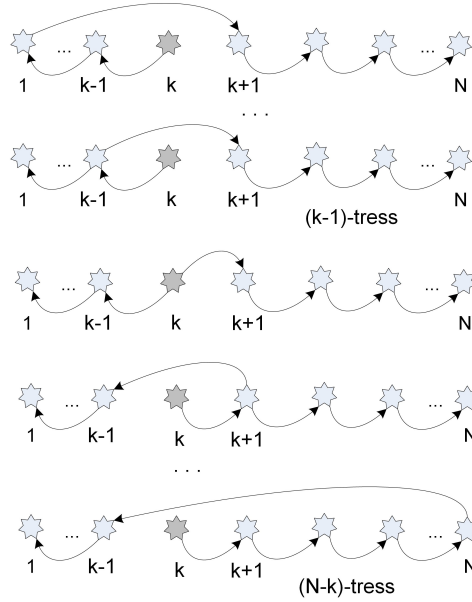


Figure 1: The set of trees for a solution of the MLB problem in L_N .

The next lemma shows that for a sufficiently large network L_N , when the number of nodes grows, the energy E_i^k , $k \in [2, N-1]$ of each node decreases.

Lemma 4. For the solution of the MLB problem given by (13)-(15) the energy of each node $E_i^k(L_\infty)$, $i \geq 1$, $k \geq 2$ is given by

$$E_i^k(L_\infty) = \left(1 + \frac{1 - \frac{E_1}{E_k}}{\sum_{r=1}^k \frac{E_1}{E_r} + \sum_{r=1}^\infty \frac{E_1}{E_r}}\right) E_1 Q_k. \quad (20)$$

Proof. The solution (13)-(15) is an equal energy solution with the energy of each node $E_i^k(L_N) = E_1 q_k^k(L_N) + E_1 Q_k$, $i \in [1, N]$, $k \in [2, N-1]$, where $q_k^k(L_N)$ is given by (14). For $N \rightarrow \infty$ the energy $E_i^k(L_N)$ has the limit (20). \diamond

3 Broadcast transmission with wireless multicast advantage

Because any data broadcasted by the k -th node of the L_N network is transmitted along some spanning tree $t^{k,r} \in V_k$, we define the wireless multicast advantage property of a broadcast data transmission for each tree of the set V_k . In the first section we defined V_k as the set of all spanning trees of the graph $\{L_N, V, E\}$ rooted at the k -th node and we assumed that edges of each tree $t^{k,r}$ were arranged in some fixed sequence. Because any WMA transmission along given tree $t^{k,r}$ is defined by some ordering of its edges $t_{i,j}^{k,r}$, we extend the set V_k by adding to it $(\alpha_r - 1)$ trees $t^{k,r,\alpha}$ obtained by reordering edges of the tree $t^{k,r}$. Any ordered tree $t^{k,r,\alpha}$, $\alpha \in [1, \alpha_r]$, where $t^{k,r,1} = t^{k,r}$ is an initial ordering (6), defines some WMA data transmission. The set of all trees $t^{k,r,\alpha}$, $\alpha \in [1, \alpha_r]$, $r \in [1, N^{N-2}]$ rooted at the k -th node we denote by \bar{V}_k . In Figure 2 there are two different orderings of a tree rooted at the second node of the L_6 network shown, which define different WMA data transmissions.

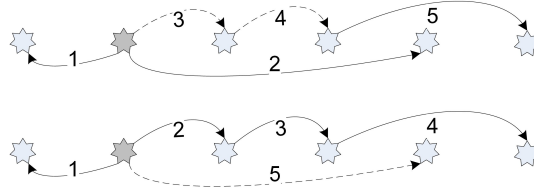


Figure 2: Two nonequivalent WMA data transmissions along the same tree in L_6 .

The numbers on the edges denote the sequence the data is transmitted along the tree and the dotted lines indicate the costless transmission.

Let us denote by $E_{i,j}(q)$ the energy consumed by the i -th node to transmit the amount q of data to the j -th node. By $U_{i,j}$ we denote the set of network nodes which are in the transmission range of the i -th node, when the data is transmitted to the j -th node, and can receive the transmitted data without additional cost of the transmitter

$$U_{i,j} = \{n \in S_N \mid E_{i,j}(q) > 0 \Rightarrow E_{i,n}(q) = 0\}.$$

From the definition of $U_{i,j}$ it follows, that the data transmitted by the i -th node to the j -th node and nodes from $U_{i,j}$ is parallel and is performed along the edges $\{t_{i,j}, t_{i,n}\}_{n \in U_{i,j}}$ of some tree t . Definition of a set $U(t_{i,j}^{k,r}) \equiv U_{i,j}^{k,r}$ for each edge of a tree $t^{k,r} \in V_k$ is equivalent to the definition of some ordered tree $t^{k,r,\alpha} \in \bar{V}_k$, $\alpha \in [1, \alpha_r]$ and some WMA data transmissions along $t^{k,r}$. It may happen that different trees $t^{k,r,\alpha}$ and $t^{k,r',\alpha'}$ from \bar{V}_k define equivalent WMA data transmissions. Figure 3 shows an ordered tree in L_6 which defines WMA data transmission equivalent to the transmission along the first tree in Figure 2.

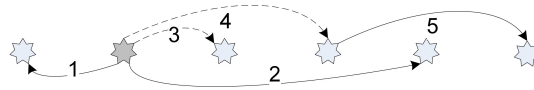


Figure 3: Parallel WMA data transmission along a tree in L_6 .

By $q_{r,\alpha}^k$ we denote the data transmitted along the ordered tree $t^{k,r,\alpha} \in \bar{V}_k$. The WMA set $U(t_{i,j}^{k,r,\alpha}) \equiv U_{i,j}^{k,r,\alpha}$ for the data transmission along the tree $t^{k,r,\alpha}$ we define as follows

$$U_{i,j}^{k,r,\alpha} = \{n \in S_N \mid E_{i,j}(q_{r,\alpha}^k) > 0 \Rightarrow \forall_{m \in S_N} E_{m,n}(q_{r,\alpha}^k) = 0\}.$$

We also assume that

$$E_{m,n}(q_{r,\alpha}^k) = 0 \Rightarrow U_{m,n}^{k,r,\alpha} = \emptyset, \quad (21)$$

which means that the WMA property cannot be applied to the data transmission for which $E_{m,n}(q) = 0$. Note, that this assumption is non-trivial for the directional antennas. The data transmission cost energy matrix for each edge of the tree $t^{k,r,\alpha}$, $E_{i,j}(t^{k,r,\alpha}) \equiv E_{i,j}^{k,r,\alpha}$ having the WMA property and satisfying (21) can be defined by the following formulas

$$E_{i_n,j_n}^{k,r,\alpha} = \begin{cases} E_{i_1,j_1}, n = 1, \\ E_{i_n,j_n}, j_n \notin \bigcup_{s=1}^{n-1} U_{i_s,j_s}^{k,r,\alpha}, n \in [2, N-1], \\ 0, j_n \in \bigcup_{s=1}^{n-1} U_{i_s,j_s}^{k,r,\alpha}. \end{cases} \quad (22)$$

The energy consumed by the i -th node, written in terms of $E_{i,j}^{k,r,\alpha}$, equals

$$E_i(q^k) = \sum_{j=1, j \neq i, k}^N \sum_{r,\alpha} q_r^{k,\alpha} t_{i,j}^{k,r,\alpha} E_{i,j}^{r,\alpha}. \quad (23)$$

The objective function of the MLB problem with WMA property has the form (4) where $E_i(q^k)$ is given by (23). The following lemma describes a solution of the MLB problem with WMA property when nodes of the L_N network use the bidirectional antennas.

Lemma 5. For the bidirectional antenna the solution of the MLB problem with the wireless multicast advantage property and $E_{i,j}$ satisfying (1) is given by the transmission tree

$$(t_{k,k-1}^k, \dots, t_{2,1}^k, t_{k,k+1}^k, \dots, t_{N-1,N}^k) \quad (24)$$

with the weights Q_k , $k \in [2, N-1]$.

Proof. Since the distance between neighboring nodes in the L_N network is equal to one, then for a bidirectional antenna the data transmitted by the k -th node, $k \in [2, N-1]$ to the $(k-1)$ -th node is delivered to the $(k+1)$ -th node without costs. In other words, for the WMA data transmission along the edges $(t_{k,k-1}^k, t_{k,k+1}^k)$ the amount Q_k of data is delivered to the both nodes $(k \pm 1) \in L_N$ with the minimal energy $E_k^k = Q_k$ consumed by the k -th node. From Lemma 3 we know that, the optimal tree to transmit the amount Q_k of data from the $(k-1)$ -th node to the nodes $i \in [1, k-2]$ of the L_N network is given by the sequence of edges $(t_{k-1,k-2}^k, \dots, t_{2,1}^k)$ and the energy utilized by each node is equal $E_i^k = Q_k$, $i \in [2, k-2]$ and $E_1^k = 0$. Similarly, the optimal tree to transmit the amount Q_k of data from the $(k+1)$ -th node to the nodes $i \in [k+2, N]$ of the L_N network is given by the sequence of edges $(t_{k+1,k+2}^k, \dots, t_{N-1,N}^k)$ and the energy utilized by each node is equal $E_i^k = Q_k$, $i \in [k+2, N-1]$ and $E_N^k = 0$. This optimal transmission tree coincides up to an ordering with the transmission tree given by (24). \diamond

The next theorem states that for the directional antennas and for the internal nodes of the network L_N , i.e. $k \in [2, N-1]$, the solution of the MLB problem with WMA property coincides with the solution given in (13)-(15).

Theorem 2. For a directional antenna the solution of the MLB problem with the wireless multicast advantage property and $E_{i,j}$ satisfying (1), for $k \in [2, N-1]$, $N \geq 3$ is an equal energy solution and it is given by (13)-(15).

Proof. The proof is basically the as the proof of Theorem 1. \diamond

The solutions of MLB problem with the wireless multicast advantage property for the first and the last node of the L_N network and for both types of antennas, the bidirectional and directional is given by Lemma 3.

4 The maximum lifetime broadcasting algorithm

The objective of the algorithm is to find an optimal weighted graph for the point-to-point broadcast data transmission which minimizes the energy consumed by the most overloaded sensor and thus maximizes the lifetime of the network S_N . In this section we assume, that the network S_N is build of N nodes and it is embedded in $d \geq 1$ dimensional space R^d . The algorithm consists of two parts. First, we determine the optimal transmission graph g^k for the data broadcasted by the k -th node in S_N . In the second step, we minimize the objective function of the MLB problem (4) with respect to the weights $\{q_r^k\}$ of the graph g^k and the constraint $\sum_r q_r^k = Q_k$. We assume, that the optimal graph g^k is a set of N subgraphs $g^{k,r}$, $r \in [1, N]$ and each subgraph is a sum of a shortest path $p^{k,r}$ from the broadcasting node to the r -th node and the minimum spanning tree $t^{k,r}$ in the graph $G_N \setminus \bar{p}^{k,r}$ rooted at the r -th node, where $\bar{p}^{k,r} = p^{k,r} \setminus \{x_r\}$, which the out-order of each node does not exceed two. The assumption, that the maximum out-order of the nodes of g^k is equal to two follows from the observation that, copying the data unnecessary overloads the sensors, and as a general rule we allow the nodes to send the same data only to the maximum of two nodes. For each $g^{k,r}$, $r \in [1, N]$ we assign a weight q_r^k which defines the amount of data transmitted along this subgraph. The objective function of the algorithmic MLB problem is given by (4), where the energy of each node $E_i(q^k) = \sum_{r,j} q_r^k g_{i,j}^{k,r}$ is expressed in terms of the optimal graph $g^k = \{p^{k,r} \cup t^{k,r}\}_r$ and its weights q_r^k .

The maximum lifetime broadcasting algorithm for the point-to-point data transmission is defined as follows:

Step 1. In the graph $G_N = \{S_N, V, E\}$ determine the set $V_{k,k}$ of ordered, minimal spanning trees rooted at the k -th node with the maximum out-order of each node equal to two. Select a one tree $t^{k,k}$ from the set $V_{k,k}$. If the tree $t^{k,k}$ is a path, then it is a solution of the algorithmic MLB problem with the weights Q_k , i.e. $q_{i,j}^k = Q_k t_{i,j}^{k,k}$, else go to the Step 2.

Step 2. In G_N determine the shortest path $p^{k,r}$ from the k -th node to each node $r \neq k$ of the network S_N . For the subgraph $G_N \setminus \bar{p}^{k,r}$, where $\bar{p}^{k,r} = p^{k,r} \setminus \{x_r\}$, determine the ordered, minimum spanning tree $t^{k,r}$, $r \in [1, N]$, $r \neq k$ rooted at the r -th node with the maximum out-order of each node equal to two. The graph, which is a sum of $p^{k,r}$ and $t^{k,r}$ we denote by $g^{k,r}$, $r \in [1, N]$.

Step 3. To each graph $g^{k,r}$ assign a weight q_r^k and find the minimum of the objective function $E(q^k) = \max_i \{E_i(q^k)\}$ with respect to the N variables q_r^k and with the constraint $\sum_{r=1}^N q_r^k = Q_k$, where $E_i(q^k) = \sum_{r,j} q_r^k g_{i,j}^{k,r}$. For determined tuple of N numbers $q_r^k = (q_1^k, \dots, q_N^k)$, which minimizes the objective function $E(q^k)$, the solution of the algorithmic MLB problem can be given by the data flow matrix $q_{i,j}^k = \sum_r q_r^k g_{i,j}^{k,r}$. \diamond

To determine the shortest path $p^{k,r}$ or the minimum spanning tree $t^{k,r}$ in a weighted graph G_N we can apply a well known graph searching algorithms. For example, the Dijkstra algorithm for finding the shortest path and the Kruskal algorithm for finding the minimum spanning tree, [6]. In general, for such algorithms elements of the weight matrix E in $G_N = \{S_N, V, E\}$ can be arbitrary non-negative numbers $E(x_i, x_j) \geq 0$, where x_i, x_j are location of the sensors in R^d . In the previous sections, we considered the MLB problem with the data transmission cost energy matrix $E(x_i, x_j)$, which defines the weight matrix E in G_N , to be a superadditive function. In dimension $d \geq 2$ the weight function matrix $E_r(\bar{a}, \bar{\lambda})$ given by (12) satisfy the superadditivity property $E_{r+r'} \leq E_r + E_{r'}$ when $r, r' \in R_+^d$ and $\forall_n a_n \geq 2$. For such weights the maximum lifetime broadcasting algorithm determines the same optimal graph $g^{k,r}$ in G_N . It means, that the only difference between solutions of the algorithmic MLB problem for a given network S_N and $E_r(\bar{a}, \bar{\lambda})$ is given by various weights q_r^k determined in the third step of the algorithm. The following lemma states that for the MLB problem in the L_N network the data transmission graph determined by the maximum lifetime broadcasting algorithm is an exact solution of the MLB problem in L_N .

Lemma 6. The solution of the MLB problem (11) determined by the MLB algorithm for the

internal nodes of the L_N network is given by (13)-(15).

Proof. Because any data transmission cost energy matrix $E_{i,j}$ of the form (12) satisfies (1) then the minimum spanning tree determined by the algorithm rooted at the k -th node ($k \neq 1, N$) weighted by (12) coincides with the spanning tree $t^{k,k}$ given by the solution of the MLB problem in Theorem 1. Each tree $t^{k,r}$, $r \neq k, r \in [1, N]$ from (13)-(15) is a sum of a shortest path $p^{k,r}$ and minimum spanning tree $\tilde{t}^{k,r}$ in the graph $L_N \setminus \bar{p}^{k,r}$. For the weights of the graph $G_N = \{L_N, V, E\}$ given by the data transmission cost energy matrix $E_{i,j}$ of the form (12) the shortest path $p^{k,r}$ between the k -th node and r -th node of L_N is given by the distance between them and the minimum spanning tree in the subgraph $L_N \setminus \bar{p}^{k,r}$, where $\bar{p}_{k,r} = p_{k,r} \setminus \{r\}$ is given by (13) and coincides with the spanning tree $\tilde{t}^{k,r}$ determined by the algorithm in $L_N \setminus \bar{p}^{k,r}$. \diamond

When the data is broadcasted by the first and the last node of the L_N network, then the optimal transmission graph determined by the MLB algorithm is a path and coincides with the solution of the MLB problem given in Lemma 3. For a sensor network S_N which forms a regular grid the weights $E_{i,j}$ are of the form (12), the minimum spanning tree is the path and the algorithm gives an exact solution of the MLB problem. For several sensor network configurations in two dimensions the MLB algorithm gave for some nodes of S_N an exact solution and for other nodes the energy level was 30% worse with comparison to the exact solution, i.e., $\frac{E(q^{alg})}{E(q^{opt})} \simeq 1.3$.

5 Conclusions

In this paper we solved the maximum lifetime problem for a point-to-point and a point-to-multipoint broadcast data transmission in one dimensional regular sensor network L_N . Based on the analytical solution of the problem in L_N , we proposed an efficient algorithm for finding a weighted graph in a sensor network of any dimension and for the point-to-point broadcast data transmission which allows to extend the network lifetime. The main difference between the MLB algorithm and the known broadcasting algorithms is that the proposed algorithm splits the broadcasted data into N parts, sends each part along different transmission trees and by this spreads more evenly the energy consumed by sensors of the network. For evenly distributed sensors in a given area in R^d one can expect that there exists an equal energy solution of the maximum lifetime broadcasting problem. For such networks the third step of MLB algorithm, in which we minimize the objective function (4), can be replaced by the problem of solving the following set of linear equations

$$\begin{cases} \sum_r q_k^r = Q_k, \\ \sum_{r,j} q_k^r (g_{i,j}^{k,r} - g_{i+1,j}^{k,r}) = 0, \quad i \in [1, N-1]. \end{cases}$$

Knowledge about existence of an equal energy solution of the maximum lifetime broadcasting problem greatly simplifies the solution of the problem. It seems, that the most interesting, unsolved problem for the presented MLB algorithm is to find a criteria for deciding whether for a given distribution of sensors in the space R^d an equal energy solution of the maximum lifetime broadcasting problem exists.

References

- [1] A. Giridhar, P.R. Kumar, Maximizing the functional lifetime of sensor networks, in IPSN 2005: Proceedings of the 4-th International Symposium on Information Processing in Sensor Networks, Piscataway, NJ, USA, 2005, IEEE Press.
- [2] T. Acharya, G. Paul, Maximum lifetime broadcast communications in cooperative multihop wireless ad hoc networks: Centralized and distributed approaches, Ad Hoc Networks 11 (2013), pp.1667-1682.

- [3] J. E. Wieselthier, G. D. Nguyen, A. Ephremides, Algorithms for Energy-Efficient Multicasting in Static Ad Hoc Wireless Networks, *Mobile Networks and Applications* 6, pp. 251-263, 2001.
- [4] C. Berge, *Graphs*, North Holland, 1989.
- [5] M. Garey, D. Johnson, *Computers and Intractability: a guide to theory of NP-Completeness*, Freeman, San Francisco, USA, 1979.
- [6] T. H. Cormen, Ch. E. Leiserson, R. L. Rivest, C. Stein, *Introduction to Algorithms*, MIT Press 2009.